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RESEARCH
ANALYSIS
CORPORATION

Preliminary Evaluation of a Range-Estimating Concept for Use with Range-Critical Infantry Weapons

by
Charles A. Bruce
Paul F. Michelsen
Robert E. Shook
Richard E. Tiller

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RAC

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DEPARTMENT OF THE ARMY
OFFICE OF THE CHIEF OF RESEARCH AND DEVELOPMENT
WASHINGTON, D.C. 20310

CRD/J

SUBJECT: RAC-TP-218, "Preliminary Evaluation of a Range-Estimating Concept for Use with Range-Critical Infantry Weapons"

TO:

1. The attached RAC technical paper is forwarded for information. Firm conclusions as to the overall value of a range estimation device utilizing matching reticle image to target cannot be drawn without further evaluation using combat type targets and signatures.

2. Reference the first sentence of <u>Results</u> (page 2), a figure of ten per cent error criterion is indicated as being the Army accepted error deviation. This figure was apparently obtained from a United States Army Materiel Command Draft Proposed Small Development Requirement for a hand held optical range finder (References, item 2, page 39). This error factor does not have Army wide acceptance; therefore, its positive use as a measurement base is suspect.

3. The concept of matching reticle image to target, while possessing certain inherent limitations, has military potential, especially in the field of light infantry antitank and antipersonnel weapons. Firm conclusions should not be drawn until after further testing with other than erect personnel targets and consideration of cost effectiveness factors involved in development of a more sophisticated sighting system for range-critical infantry weapons.

FOR THE CHIEF OF RESEARCH AND DEVELOPMENT:

Incl
as

Robert B. Bennett
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COMBAT ANALYSIS DEPARTMENT
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McLEAN, VIRGINIA

FOREWORD

In July, August, and November 1965 a series of three small field experiments was conducted to evaluate a new concept reticle for range estimation with low-velocity high-trajectory infantry weapons. The concept utilizes the stadiametric principle but does not require the operator to know or index into the instrument the target size.

On the basis of expressed Army interest the concept has been proposed for Military Potential Testing.

Philip H. Lowry
Head, Combat Analysis Department

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Problem

To evaluate a concept reticle for range estimation with low-velocity high-trajectory individual infantry weapons.

Background

Visual range-estimation errors are approximately 25 percent of the range.[1] The Army's Small Development Requirement for a rangefinder states that the range-estimation error should be no more than 10 percent of the range.[2] The recently developed family of light individual infantry weapons such as the M79 (40-mm grenade launcher), the M67 (90-mm recoilless rifle), and the M72 (66-mm folding-fin rocket) are low-velocity weapons that require accurate range estimation to be effective.

Discussion

Test Bed

The test bed for the concept reticle was a modified M17 elbow telescope mounted on a tripod. The reticles displayed various arrays of range-calibrated soldier silhouettes for comparative matching of silhouette to target. The device uses the stadiametric principle but does not require the operator to know or index into the instrument the target size.

Procedure

The program was comprised of three separate phases employing RAC personnel as subjects. A preliminary phase was conducted in July 1965 to establish the ability of observers to detect 20 percent size differences in high-contrast targets. An initial reticle phase was conducted in August 1965 with three reticle patterns:

(a) Six evenly spaced silhouettes proportionally increasing in size from left to right arrayed on a clearly defined horizontal line.

(b) Six evenly spaced silhouettes proportionally decreasing in size from left to right arrayed at an angle of about 30 deg to give an impression of depth in the reticle.

SUMMARY

(c) Five silhouettes proportionally different in size individually displayed on five reticles.

The second reticle phase was conducted in November 1965 to examine the potential of the concept as a ranging weapon sight with two reticle patterns:

(d) Six silhouettes arrayed in line proportionally decreasing in size from top to bottom, a pattern resembling that which would match the ballistics of a selected high-trajectory weapon.

(e) Six silhouettes arrayed in a staggered offset pattern and spaced similarly to the in-line reticle silhouette.

In the initial and second phases both stationary human targets and fleeting human targets were employed; however, greater emphasis was placed on fleeting targets in the second phase. In all phases the targets were distributed at ranges of from approximately 240 to 550 m.

Results

In the initial phase the precision of the silhouette-to-target comparisons for stationary human targets was well below the required 10 percent error criterion for Army acceptance. The standard deviations of the various patterns of silhouette array of the initial test varied from 5.0 to 7.2 percent error of range estimation. In only pattern b (silhouettes arrayed horizontally at a 30-deg angle) did a significant difference in precision appear between the ability to match target to silhouette and the ability to interpolate the target between silhouettes.

For fleeting-target trials of the initial phase, standard deviations from 1.4 to 7.8 percent error of range estimation existed for the four target positions examined. A significant difference between target positions was indicated by the data of pattern a array (horizontally arrayed silhouettes on a horizontal reference line).

In the second phase the precision of percentage error of range estimation was inconsistent with regard to target position or range of target, but the standard deviation was less than 10 percent in all but one case for the stationary target. For several target positions, statistically significant (5 percent level) differences were indicated between patterns of the vertically arrayed silhouettes, although these differences did not consistently indicate the pattern yielding the greater precision. For the fleeting-human-target trials, standard deviations of percentage error of range estimation varied from 6.0 to 13.3 percent for the target positions investigated. With the exclusion of one apparently erroneously recorded observation with error of 70 percent, precision of percentage error of range estimation between and within matches and interpolations of targets to silhouettes were not significantly different at the 5 percent level. The overall standard deviation of the percentage of error of range estimation employing the vertical in-line silhouette reticle was 10.5 percent for the fleeting target in the second phase.

SUMMARY

Conclusion

The concept of matching reticle image to target appears to provide a technique for acceptably precise range estimation against human targets in the range interval of low-velocity high-trajectory infantry weapons.

Preliminary Evaluation of a Range-Estimating Concept for Use with Range-Critical Infantry Weapons

INTRODUCTION

Problem

To evaluate a concept reticle for range estimation with low-velocity high-trajectory individual infantry weapons.

Background

Abundant experimental and experiential data indicate that visual range-estimation error is the principal limiting factor in the achievement of acceptable first-round hit probability with low-velocity high-trajectory individual weapons.[3–5] The magnitude of this error is generally recognized to be approximately 25 percent of the range. A representative source of support for this value is found in field studies by the Canadian Army Research and Development Establishment associated with the development of an 81-mm recoilless rifle. The standard deviation of the error for visual estimation of moving tank targets exposed at ranges of from 300 to 900 m in a simulated offensive situation was approximately 140 m.[6]

In response to the need for a device that would introduce minimum training requirements and provide simplicity, speed of operation, light weight, and accuracy, candidate devices have been subjected to service testing since the mid-1920's.[2] Coincident image[7] and stadiametric[8,9] units have been designed and tested, but results in the hands of average troops have been disappointing. An experiment utilizing troops from the Infantry Training Center, Ft Jackson, S. C., compared the T40 coincident-image range finder and a prototype stadiametric device with visual range estimation.[1] High-contrast and low-contrast silhouette targets, moving and disappearing personnel, and vehicles were exposed under a variety of terrain and light conditions at ranges of approximately 90 to 400 m. On stationary silhouette targets the greatest accuracy with selected men was 12 percent for the T40, 19 percent for the stadiametric unit, and about 25 percent for visual estimation. To achieve the accuracy of a 3-rd fire adjustment, a ranging device must not yield an error in excess of 12 percent.[1,10] This level of accuracy does not realize the potential of the antipersonnel or antitank warheads of the recently developed family of range-critical light individual infantry weapons in Table 1.

As is evident from the several listed characteristics of the weapons in Table 1, range-estimation error is a principal cause of low first-round hit probability of low-velocity high-trajectory weapons. In addition the need for firing 3 rds to achieve even this accuracy introduces the tactically undesirable problem of disclosure to the enemy.

The concept under evaluation utilizes the stadiametric principle but does not require the operator to know or index into the instrument the target size. In contrast to the usual stadiametric units, it utilizes as the reference index

a scale familiar to the average soldier, i.e., the silhouette of an average man. Its basic components are a low-power (2–3x) wide-field monocular equipped with a reticle that presents a graduated series of outlined images of a soldier, corresponding in size to selected range intervals from 100 to 500 m. The operator is required only to match the image of appropriate size to the target or

TABLE 1

Typical Low-Velocity High-Trajectory Individual Infantry Weapons

Weapon	Projectile weight, lb	Muzzle velocity, fps	Effective range, m
M79, 40-mm grenade launcher	0.5	250	150 (point targets) 350 (area targets)
M67, 90-mm recoilless rifle	6.8	700	450
M72, 66-mm folding-fin rocket	4.5[a]	500	225
M20, 3.5-in. rocket launcher	9.0	334	275 (point targets)

[a]Including disposable launcher.

to interpolate between images. If for example the target is an M60 tank, the user selects the image that places man and tank in proper size relation, and then reads the range from the numerical scale on the reticle. The concept exploits an individual's natural ability to recognize perspective relations, and an error therefore requires a "violation of perspective."

Scope

Inasmuch as the objective of this study was to evaluate a concept rather than a range-finding instrument, the scope was restricted to a demonstration of the feasibility and value of this concept. No attempt was made to construct a range finder; the experimental device employed simply measured the precision with which the reticle-to-target matching technique could be performed on range-calibrated stationary and fleeting targets. Quantitative data derived from a small field experiment were translated into measurements of range-estimating precision.

No attempt was made to examine analytically the 10 percent error criterion for Army acceptance. This value for required precision was simply accepted as a guideline for concept performance. Had the concept not met this requirement, it would have been discarded.

Limitations and Assumptions

Because the device employed in this evaluation was not optically perfect or accurately calibrated, the data reflect the precision with which the range-determining concept can be employed, rather than the accuracy with respect to any particular range. On this basis it is therefore assumed that a perfectly calibrated instrument would have yielded equally precise values with similar distributions around the true ranges.

THE DEVICE

To evaluate the concept quantitatively an Army 8× elbow telescope (M17) normally used on cinetheodolites was modified (see Fig. 1). This particular unit was selected for several reasons: it was readily available, it offered a

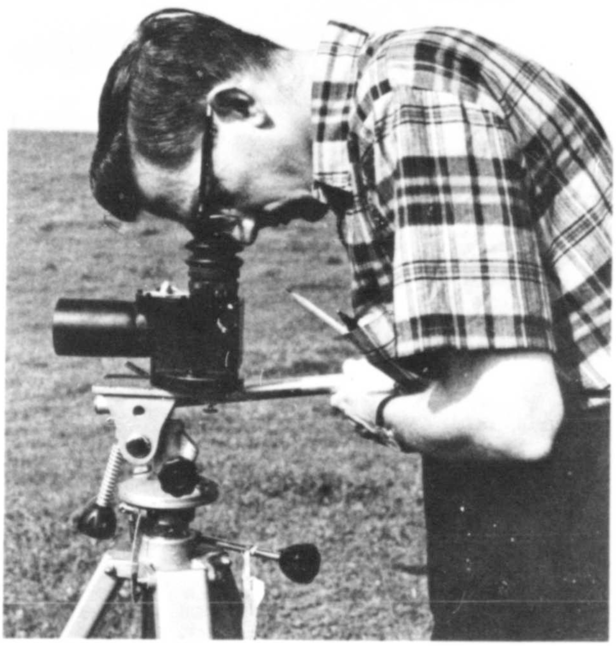


Fig. 1—The Modified US Army 8× M17 Elbow Telescope

means of installing seven reticles in place of its filter wheel, the optical design provided an erected image, and its eyepiece furnished a magnified view of the image reticle as well as the focal plane. The original 300-mm objective lens was replaced by a lens of 135-mm focal length. This resulted in an instrument of 4.8× with a linear field of view of 64 m at 500 m.

Scaled line drawings of the reticle silhouettes differing in size by 20 percent (established by the preliminary test) were photographed. These were reduced to approximately the correct reticle size and printed on high-contrast glass photographic plates that were then cut to the size of the filter wheel openings.

In the initial phase the device housed three different patterns of reticles. As seen in Fig. 2 pattern a presented six silhouettes on a clearly defined horizontal reference line, pattern b on an angle of 30 deg from the horizontal to create the impression of depth perspective, and pattern c presented a series

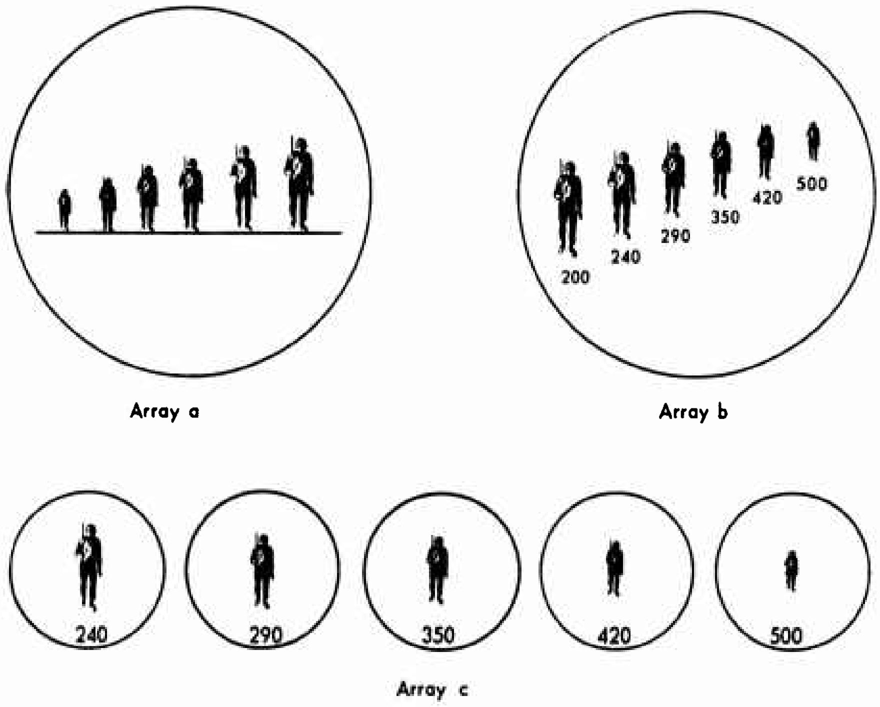
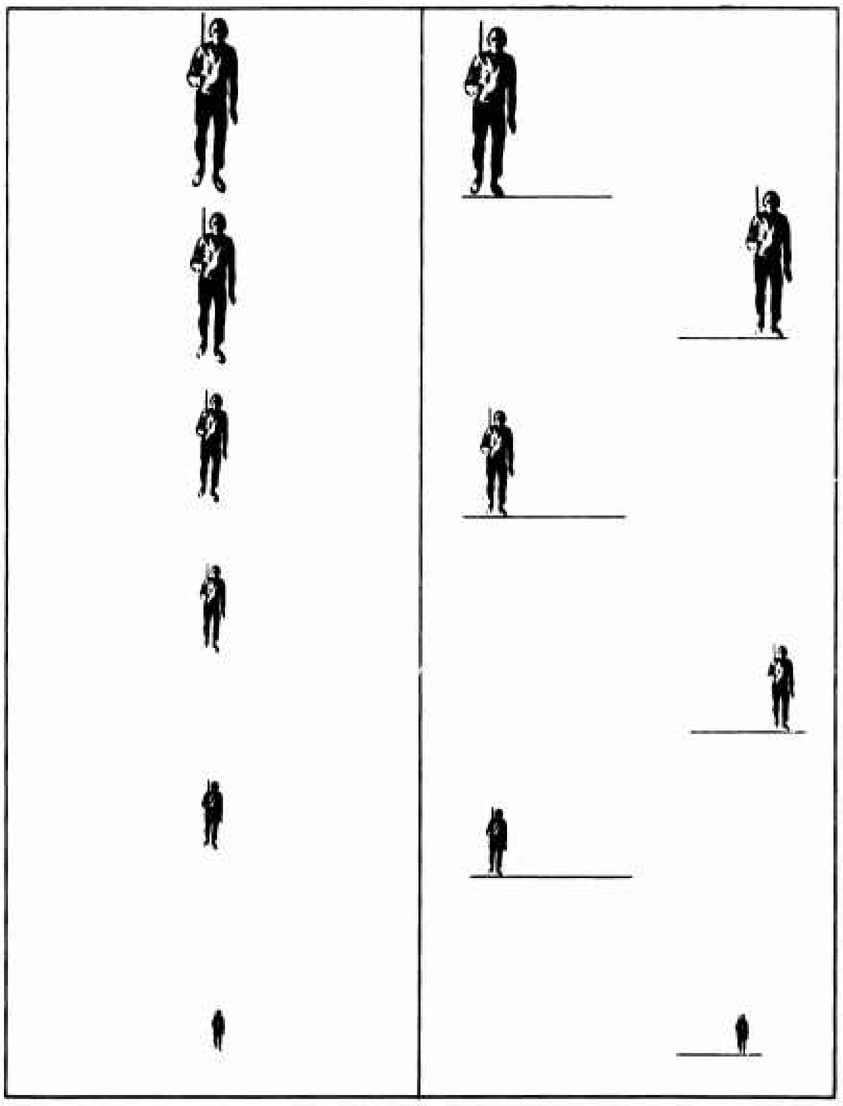


Fig. 2—Three Reticle Patterns of Horizontally Arrayed Silhouettes Used in the Initial Phase

of individual silhouettes that required the observer to rotate the indexing knob to select the appropriate silhouette.

In the second phase,consideration was given to the application of the basic concept to a ranging weapon sight, and the reticle images were presented in a vertical pattern, simulating the pattern that would match the ballistics of a low-velocity high-trajectory hand-held weapon. Two styles of vertical silhouette array, d, in line, and e, offset, were investigated (Fig. 3). Ranging procedure was basically the same, differing only in vertical (instead of horizontal) adjustment of the device, and presenting the additional requirement of placing the selected silhouette at the intersection of two range-calibrated reference points for a series of photographs that were made concurrently with the observation.

Also, in the second phase two 16-mm instrumentation cameras were mounted coaxially beneath the device to provide a method for recording the accuracy of observations. The 50-mm lenses used on the cameras resulted in an angle of view of 11 deg, which was larger than the associated reticle field angle. In employing this camera system the ratio of film size to angular mils for the recorded images presented an accuracy greater than was required for the purposes of the test. Geometric analysis of the projected images of these films, using the distance from the reference points to the film edge, provided a technique for converting the data to actual range-estimation errors. This conversion is described in the section "Data Collected."



In-line array d

Offset array e

Fig. 3—Two Reticle Patterns of Vertically Arrayed Silhouettes Used in the Second Phase

DESCRIPTION OF FIELD EVALUATION

Three separate field phases were conducted: preliminary phase, initial phase, and second phase.

The preliminary phase was conducted with unaided vision in early July 1965. It consisted of a comparison of a 6-ft silhouette with nine different-sized silhouettes of a soldier in field dress. The nine figures, painted flat black, ranged in size from 20 percent larger to 20 percent smaller than a 6-ft figure

(i.e., ± 20 percent, ± 10 percent, ± 5 percent, ± 2½ percent, and the 6-ft figure). The field exercise was merely an a priori random-ordered presentation of the figures at various ranges (100 to 500 m) from the subjects (observers). Data from the preliminary phase of the program established the ability of an observer to detect size differences of 20 percent with unaided vision.

The initial phase (August 1965) and the second phase (November 1965) were conducted with the device and are described in the following sections.

Field Layout and Experimental Subjects

The experimental site was located in an open, gently rolling pasture near Herndon, Va.

In the second phase, triangular field reference markers were placed in view downrange for photoinstrumentation calibration. One marker, pointing to the left, was placed at about 250 m. Another marker was placed at about 800 m downrange and pointed vertically down.

Five members of the RAC technical staff who were thoroughly familiar with the experimental objectives served as subjects for target-silhouette comparisons during the field exercises.

Design

The evaluation was designed to investigate the precision with which silhouette-target comparison could be performed with the various reticle silhouette displays. Ranges of silhouette-target matches as well as interpolations of silhouette-target comparisons for stationary targets and for fleeting targets were investigated. Each of the five subjects made one observation (comparison) at each range for each reticle array pattern. The targets presented for silhouette comparison were men of average height (the average height of a US male 20 to 24 years of age is 5 ft 8 in.).[11] The following is the text of printed instructions given to the subjects before the trials.

OBSERVER INFORMATION AND INSTRUCTIONS

In this field test you are asked to match the reticle image of device with the target figures in the field. You will not be told the ranges of the targets in the field. You are not to estimate the range to figure image of the reticle. In some cases you may not obtain a match of the target and the reticle image. In such cases, mark your data form accordingly with appropriate indications between representations on your form. You are asked to use the marking notation given on the example passed among you with the encircled number corresponding to the announced run.

It is important to point out to you that the numbers appearing under the reticle images are not ranges. These numbers are only for identification of the image match you find for ease in recording your results on the data form.

Your cooperation in not discussing your observations and remarking about the device during the test is required. You are asked to remain facing away from the target in the field before and after you record your observation on the data form. Do not hesitate to include your remarks or observations on the back of the data form throughout the field testing. A debriefing session will be conducted at the finish of the field research.

Procedure

Initial Phase. Two men standing erect served as the stationary target for evaluation of the horizontal reticle that presented the three arrays of silhouettes described previously in Fig. 2. Ninety silhouette-target comparisons were conducted at six ranges: three corresponding to silhouette-target matches and three corresponding to interpolations between two reticle silhouettes. Observations using the three reticle patterns were conducted in varied order.

For the fleeting-human-target exercises one man served as the target. At the time a subject observer was viewing through the appropriate reticle (patterns a or b), the target would stand up and move quickly out of sight, exposed for a distance of about 20 m and in view for 5 to 6 sec. The trials were conducted at two ranges of silhouette-target matches and one range of size interpolation between two silhouettes. Reticle pattern c was not evaluated in the fleeting-target trials.

The individual trials conducted for both the stationary- and fleeting-human silhouette-target comparisons of the initial phase are listed in Table 2. The five experimental subjects made independent observations of silhouette-target comparisons in random order on the various trials.

TABLE 2

Silhouette-Target Comparisons in Initial Phase

Trial	Target position, m	Target-silhouette comparison	Array pattern	Observer order				
				1st	2d	3d	4th	5th
Stationary Human Target								
1–5	240	Matched	b	2	1	5	4	3
6–10	240	Matched	a	5	4	3	1	2
11–15	240	Matched	c	5	3	2	1	4
16–20	358	Matched	a	3	5	1	4	2
21–25	358	Matched	b	4	3	5	2	1
26–30	358	Matched	c	3	4	2	1	5
31–35	561	Interpolated	a	3	4	5	1	2
36–40	561	Interpolated	c	1	3	2	4	5
41–45	561	Interpolated	b	2	4	3	1	5
46–50	325	Interpolated	c	1	3	4	2	5
51–55	325	Interpolated	a	1	2	3	5	4
56–60	325	Interpolated	b	1	4	2	5	3
61–65	460	Interpolated	b	4	2	3	5	1
66–70	460	Interpolated	c	5	2	4	1	3
71–75	460	Interpolated	a	4	1	2	5	3
76–80	492	Matched	c	4	5	2	1	3
81–85	492	Matched	b	1	5	4	2	3
86–90	492	Matched	a	3	4	5	1	2
Fleeting Human Target								
1–5	492	Matched	a	1	2	3	4	5
6–10	428	Matched	a	3	4	5	1	2
11–15	460	Interpolated	b	5	1	2	3	4
16–20	492	Matched	b	5	4	3	2	1

Second Phase. In this phase the reticles with vertically arrayed silhouettes (Fig. 3) were used. Only the in-line array was employed against fleeting targets. The trials were similar to the initial phase except for the addition of photoinstrumentation as a possible technique for simpler and more accurate data collection. The individual trials conducted for both the stationary- and fleeting-human-silhouette-target comparison of the second phase are listed in Table 3.

TABLE 3

Silhouette-Target Comparisons in Second Phase

Trial	Target position, m	Target-silhouette comparison	Vertical array pattern	Observer order				
				1st	2d	3d	4th	5th
		Stationary Human Target						
1–5	300	Matched	d	4	2	1	3	5
6–10	400	Interpolated	d	1	4	2	3	5
11–15	511	Matched	d	5	3	1	4	2
16–20	272	Interpolated	d	5	1	4	2	3
21–25	436	Matched	d	2	3	1	5	4
26–30	536	Interpolated	d	3	1	5	2	4
31–35	511	Matched	e	4	2	3	5	1
36–40	436	Matched	e	2	4	1	3	5
41–45	272	Interpolated	e	1	2	5	4	3
46–50	400	Interpolated	e	3	1	4	2	5
51–55	300	Matched	e	5	3	2	1	4
56–60	436	Interpolated	e	3	4	1	5	2
		Fleeting Human Target						
1–5	436	Matched	d	3	1	4	2	5
6–10	272	Interpolated	d	4	5	2	1	3
11–15	400	Interpolated	d	1	3	5	4	2
16–20	300	Matched	d	2	5	1	3	4
21–25	511	Matched	d	5	4	2	3	1

Field calibrations (by a controller) of the photoinstrumentation were made for translation of the film-recorded observations to actual range-estimation data. The calibration consisted of the following procedure before each series of target-silhouette observations at a specific range:

(a) The device was aligned with the intersection of the two field reference markers at the foot of the silhouette applicable to the target range to be evaluated or at the point of interpolation between applicable silhouettes.

(b) A series of photographs were made with the coaxially mounted cameras.

(c) The device was misaligned to avoid transfer of knowledge to the observer.

Collected Data

On the stationary-target trials the subject was timed by a stopwatch from the time he commenced to the time he completed his observation. On the fleeting-target trials the time from target appearance to time of disappearance was recorded. Time required to record data was not included.

Target ______________

Observer's Name ______________________ Date ______________

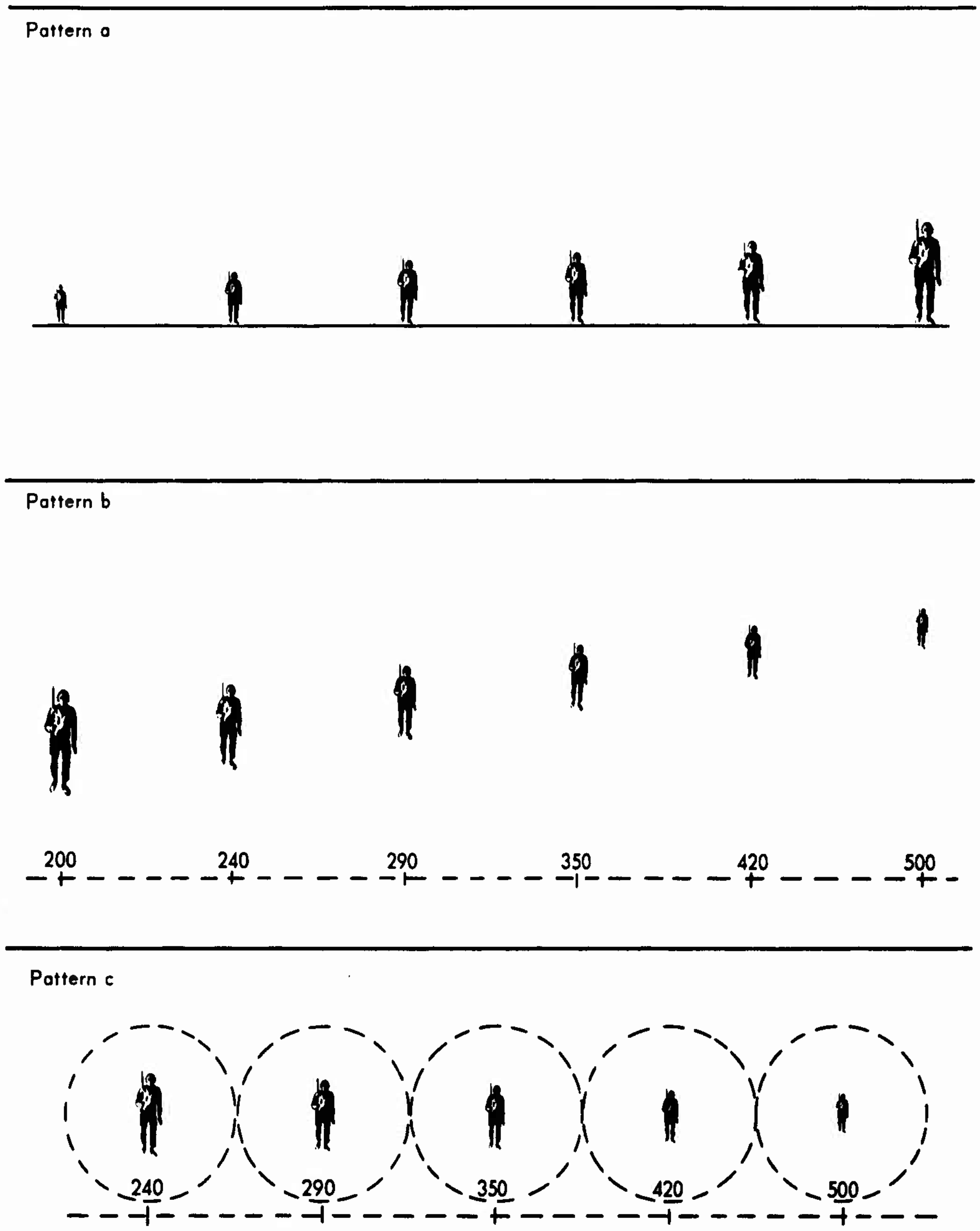


Fig. 4—Data Form for Horizontally Arrayed Silhouettes in Initial Phase

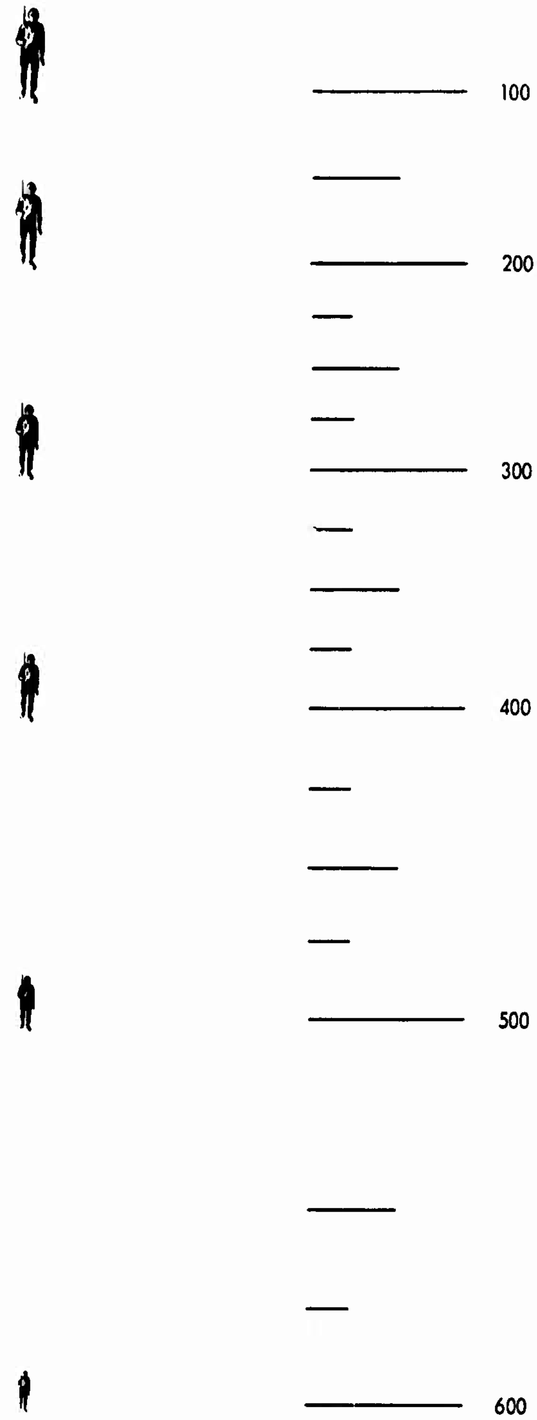


Fig. 5—Data Form for In-Line Pattern of Vertically Arrayed Silhouettes in Second Phase

During all trials each subject recorded his silhouette-target comparative selection on data forms that pictorially presented the reticle. A mark identified by a circled number corresponding to the trial being conducted was made on the form with an arrow at the point corresponding to the subject's selected reticle-silhouette match or interpolation. The data form of the initial phase is shown in Fig. 4. Figures 5 and 6 show the data forms for the in-line and offset vertical arrays used in the second phase.

The data recorded on these forms were linearly translated to range estimation by the equation

$$R_E = R_C + \sum_{i=1}^{n+1} P_i (R_{C_{i+1}} - R_{C_i})$$

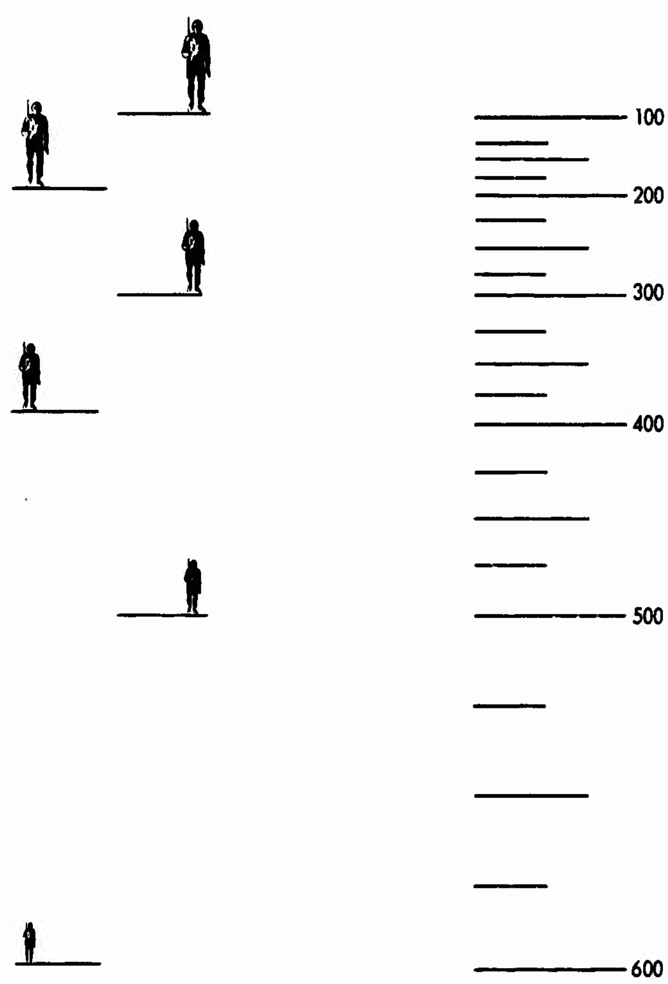


Fig. 6—Data Form for Offset Pattern of Vertically Arrayed Silhouettes in Second Phase

where R_E = the range estimation in meters

R_C = the measured calibration range of silhouette-target match in meters

$(R_{C_{i+1}} - R_{C_i})$ = the difference of the calibrated ranges of two consecutive reticle silhouettes in meters

$$R_{C_{i+1}} > R_{C_i}$$

P_i = the proportional linear increment from R_{C_i} to $R_{C_{i+1}}$ and is positive if overestimation occurs and negative if underestimation occurs

$$P_i \leq 1$$

and,

n = the number of silhouettes between the target range (where range of target = R_E) and the subject's recorded comparison

The percentage error of estimated range was calculated by the equation

$$\text{Percentage error} = 100\,(R_E - R_M)/R_E$$

where R_M is the measured target-position range in meters, and R_E is the range estimation in meters.

In the second phase camera data were collected for each trial. The film data were developed by measuring from the photographic image of the field reference marker to the top edge of the frame and translating these measurements to actual ranges from calibration curves. These curves were plotted from filmed calibrations of the silhouettes and the measured calibration ranges of the silhouettes. The curves are presented in App A.

RESULTS

The device used to demonstrate the concept was not an optically perfect or fully calibrated instrument. The calculations therefore do not indicate the accuracy with which ranging could be performed, but they indicate the precision with which the concept can be employed. The standard deviations about the estimated range provide a more realistic measure than the absolute error from measured target position. The precision (dispersion or scattering) of the observations is of primary importance and is presented as the standard deviations. The accuracy of the device (measured as the mean) is of little importance since the device itself was not fully calibrated. However, complete tabulation of means as well as standard deviations are given in App A.

Findings of both the initial and second phases are restricted to precision of percentage error of estimation range of three major categories: (a) the ability of the operators to perform a target-to-silhouette match, (b) the ability of the operators to perform an interpolation of the target to reticle silhouette, and (c) the differences of precision achieved with various reticle silhouette arrays.

Initial Phase

Stationary Human Target. The information of silhouette-target comparison as collected on the individual subject's data form was linearly translated to range in meters by the procedure described previously.

The precision of percentage error of estimation of the various ranges from 240 to 461 m (investigated in the initial phase of the field research) exhibited no significant difference (variance between ranges was not significantly different at 0.05 level). Furthermore, no significant difference (0.05 level of significance) was found in the variance from the data of several reticle arrays evaluated in the initial phase. The statistical tests for homogeneity of variance are given in App B.

However, when the precision of the comparisons of target-to-silhouette matches are compared with target-to-silhouette interpolations, statistically significant differences were exhibited. As is pointed out in the standard deviations of Table 4, the percentage errors of estimation of silhouette matches are significantly less precise than silhouette interpolations for reticle patterns b and

TABLE 4

Standard Deviations of Percentage Error of Range Estimation in Initial Phase

(Stationary human targets)

Silhouette array pattern	Observation		Variance from 0.05 level of significance
	Match	Interpolate	
	Standard deviation, %		
a	7.4	5.4	No
b	6.3	3.1	Yes
c	9.4	3.3	Yes

c. The variance of the data from each style of reticle exhibited greater variation in the target-to-silhouette matches than in the target-to-silhouette interpolations. The results of pattern a (horizontally arrayed silhouettes on a reference line) observations showed no significant difference in the precision of the operator to match or interpolate target and silhouette.

It may also be noted in pattern b that although a statistically significant difference exists between matching and interpolating target to silhouette, the precision of pattern b results is not significantly greater than that of a and c. The overall standard deviations of the three silhouette arrays are given in Table 5. No significant difference exists in the precision among reticle styles.

The average times required by the observers to make selections of silhouette-target comparisons with the three patterns of horizontal array of silhouette presentations are given in Table 6. In no case was the observer hurried or rushed to make a decision. For that reason the absolute times have little meaning; however, relative to the various patterns the differences in the overall averages show that pattern c required about 10 sec longer to use

TABLE 5

Overall Standard Deviations of Percentage Error of Range Estimation in Initial Phase

(Stationary human targets)

Silhouette array pattern	Standard deviation, %
a	6.6
b	5.0
c	7.2

TABLE 6

Average Observation Times in Initial Phase

(Stationary human targets)

Pattern	Observer 1	Observer 2	Observer 3	Observer 4	Observer 5	Average for all observers
	Time, sec					
a	33.5	39.5	23.0	29.0	28.2	30.6
b	42.0	35.2	21.8	20.2	32.8	30.2
c	55.2	50.8	29.2	36.5	38.2	41.5

than a and b. The increased time reflects the additional requirement to rotate the indexing knob for target-silhouette comparisons.

Fleeting Human Target. As is explained in the preceding section, the fleeting target was exposed to each subject observer for 5 to 6 sec on each trial of the initial phase.

The dispersion of the percentage error of range estimation of fleeting targets in the initial phase indicated a significant difference in the variance between target ranges of pattern a observations. The precision of the pattern b results was unaffected by range. Table 7 gives the standard deviations of

TABLE 7

Standard Deviations of Percentage Error of Range Estimation in Initial Phase

(Fleeting human targets)

Silhouette array pattern	Standard deviation, %
a	
492-m target position	1.4
428-m target position	7.8
Overall	10.8
b	
460-m target position	5.7
492-m target position	4.6
Overall	6.6

percentage error of range estimation for fleeting targets in the initial phase. The overall estimation of precision (variance) of the two silhouette arrays examined for fleeting-target silhouette-target comparison are not significantly different.

Second Phase

Stationary Human Targets. In the second phase two reticles with vertically arrayed silhouettes were examined (Fig. 3).

The precision of the percentage error of range estimation was inconsistent over target range, but less than 10 percent in all cases for the in-line array. The standard deviations of percentage of error for target positions for both arrays are listed in Table 8 with indicated significant differences in variance.

TABLE 8

Standard Deviations of Percentage Error of Range Estimation in Second Phase

(Stationary human targets)

Target position, m	Array		Variance from 0.05 level of significance
	In line	Offset	
	Standard deviation, %		
272	2.0	0.0	Yes
300	5.8	5.9	—
400	9.3	8.7	—
436	3.9	6.1	—
511	7.1	2.2	Yes
536	0.6	11.3	Yes

The data of percentage error of range estimation indicated significant differences in precision between the target positions within array. The precision of percentage error was neither consistently greater nor consistently less because of range. In the data resulting from the vertical in-line array the standard deviations of the percentage error of range estimation varied from 0.6 percent at the farthest target position (536 m) to 9.3 percent at the 400-m target positions; in the offset array the standard deviations ranged from 0.0 percent at the nearest target position (272 m) examined to 11.3 percent at the 536-m target position. Significant differences were also exhibited between the ranges of target-to-silhouette interpolations and between the ranges of target-to-silhouette matches.

The increasing linear distance between silhouettes arrayed to simulate the elevation of a typical low-velocity high-trajectory weapon as a fraction of range may be the cause of the lack of consistency in the precision of the results; however, it seems that precision should decrease as range increases, which is not the case in the data from the second phase of stationary targets.

As in the initial phase of the program, the time required for each experimental subject to select his choice of silhouette-target comparison was measured in the second phase. As expected the individual time requirements differed considerably and in no case was the subject hurried or rushed to make a decision. Table 9 gives the average times of the five observers. In all but one case (Observer 2) the trials employing the vertical in-line reticle (conducted before the offset trials) required more time than the trials using the offset silhouette array. This difference can be accounted for by the increased familiarity of operation gained before the offset-array trials.

TABLE 9

Average Observation Times in Second Phase

(Stationary human targets)

Vertical array	Observer 1	Observer 2	Observer 3	Observer 4	Observer 5	Average for all observers
	Time, sec					
In line	88	39	32	69	30	52
Offset	44	39	26	43	29	36

Fleeting Human Targets. The standard deviations of percentage error of range estimation ranged from 33.6 percent at the 511-m target position to 6.0 percent at the 400-m target position.

At the 511-m target position one range error (underestimated) of 70 percent accounted for much of the resulting variance at that position. Excluding the 70 percent error observation and considering the four remaining observations, the standard deviation of percentage error of range estimation is 6.1 percent (reduced from 33.6 percent). Since it is quite likely that such an error of 70 percent (twice the amount of error of any other observation) is erroneous data, it has been excluded from the calculations of the following results given in Table 10.

With the exclusion of the 70 percent error data point at the 511-m target position, no significant difference was found in the precision of the data between the various target positions. The standard deviations of the percentage error ranged from 13.3 to 6.0 percent. Precision between and within matches and interpolations of targets to silhouettes was not significantly different at the 5 percent level. The overall standard deviation of the percentage of error of range estimation employing the vertical in-line reticle was 10.5 percent for the fleeting targets in the second phase.

Correlation on Film–Data-Form Results. The overall results of the range estimation data reduced from film exhibited an average difference of 1.1 percent of the measured range when compared with the translated ranges reduced from the observers' individual data form. For the stationary human-target trials of the second phase of the program this average difference was 0.2 percent of the measured range. In the fleeting-target trials the average

TABLE 10

Standard Deviations of Percentage Error of Range Estimation in Second Phase

(Fleeting human targets; in-line array)

Target position, m	Target-silhouette comparison	Standard deviation, %
272	Interpolated	6.9
300	Matched	13.3
400	Interpolated	6.0
436	Matched	6.8
511[a]	Matched	6.1
Overall	Matched	11.8
Overall	Interpolated	6.1
Overall matched and interpolated		9.9

[a]Excluding 70.3 percent (underestimated) error.

difference was 3 percent of the measured range. That the difference between film data and form data should be larger for fleeting targets is understandable, since the observer made a less accurate target-silhouette comparison and consequently a less accurate adjustment of the device with respect to the target complex downrange in the short period of time that the target was in view in the fleeting trials.

DISCUSSION

In the concept evaluation described in this report, RAC personnel served as subjects. No conditions of stress or combat were imposed on the subjects. Furthermore, the device was mounted on a tripod during the evaluation. In operational configuration the device would be hand-held.

A further evaluation should (a) utilize Regular Army personnel as subjects, (b) impose time restrictions on the comparative matching of the subjects, (c) employ personnel targets of varying heights, (d) employ nonpersonnel as well as personnel targets, (e) include additional ranges both 200 m shorter and longer than the span investigated herein, (f) examine more fully the employment of the device against fleeting targets, and (g) utilize a hand-held device for silhouette presentation to the observer.

The concept also possesses potential for other applications. One such application is the employment of the concept to introduce the estimation of range in night-vision devices such as the image intensifier. In the sight of the image intensifier, perception of depth or range is greatly reduced. The concept technique does not require recognition of depth, but relies on recognition of size relation between target and reticle image. Another application would combine ranging and aiming by employing a reticle pattern that would introduce elevation. Such an application would require that the reticle be tailored to the specific weapon's external ballistics.

CONCLUSION

The concept of matching reticle image to target appears to provide a technique for acceptably precise range estimation against human targets in the range interval of low-velocity high-trajectory infantry weapons.

Appendix A

RANGE-ESTIMATION DATA, PERCENTAGE

ERRORS, AND FILM-READING CURVES

Tables

Figures

TABLE A1

Range-Estimation Data for Horizontally Arrayed Silhouettes in the Initial Phase

(In meters)

Array pattern	Target position	Observer 1	Observer 2	Observer 3	Observer 4	Observer 5
		Stationary Human Target				
a	240	272	251	254	286	255
	325	356	363	336	336	417[a]
	358	405	430	436	385	358
	460	498	498	481	469	480
	492	498	498	465	474	494
	561	597	524	585	534	531
b	240	271	293	286	271	282
	325	362	352	340	599[a]	339
	358	403	405	434	388	385
	460	520	494	484	494	492
	492	486	474	523	532	532
	561	604	626	604	574	650
c	240	278	316	312	308	304
	325	344	333	370	354	348
	358	406	416	372	382	367
	460	529	500	510	525	500
	492	474	555	484	484	572
	561	600	600	596	565	596
		Fleeting Human Target				
a	492	498	498	498	486	486
	428	589	540	456	510	540
b	460	436	500	458	436	473
	492	436	436	480	467	448

[a]Excluded from calculations as recording error.

TABLE A2

Percentage Error of Range Estimation for Horizontally Arrayed Silhouettes in the Initial Phase

Array pattern	Target-silhouette comparison	Target position, m	Observer 1	Observer 2	Observer 3	Observer 4	Observer 5
			Error, %				
			Stationary Human Target				
a	Matched	240	11.8	4.4	5.5	16.1	5.9
	Interpolated	325	8.7	10.5	3.3	3.3	22.1[a]
	Matched	358	11.6	16.7	17.9	7.0	0.0
	Interpolated	460	7.6	7.6	4.4	1.9	4.2
	Matched	492	1.2	1.2	−5.8	−3.8	0.4
	Interpolated	561	6.0	−7.1	4.1	−5.0	−5.6
b	Matched	240	11.4	18.1	16.1	11.4	14.9
	Interpolated	325	10.2	7.7	4.4	45.7[a]	4.1
	Matched	358	11.2	11.6	17.5	7.7	7.0
	Interpolated	460	11.5	6.9	5.0	6.9	6.5
	Matched	492	−1.2	−3.8	5.9	7.5	7.5
	Interpolated	561	7.1	10.4	7.1	2.2	13.7
c	Matched	240	13.7	24.0	23.1	22.1	21.0
	Interpolated	325	5.5	2.4	12.2	8.2	6.6
	Matched	358	11.8	13.9	3.8	6.3	2.4
	Interpolated	460	13.0	8.0	9.8	10.4	8.0
	Matched	492	−3.8	11.4	−1.6	−1.6	14.0
	Interpolated	561	6.5	6.5	5.9	0.7	5.9
			Fleeting Human Target				
a	Matched	492	1.2	1.2	1.2	−1.2	−1.2
	Matched	428	27.3	20.7	6.1	16.1	20.7
b	Interpolated	460	−5.5	8.0	−0.4	−5.5	2.7
	Matched	492	−12.8	−12.8	−2.5	−5.4	−9.8

[a]Excluded from calculations as recording error.

TABLE A3

Measures of Range Estimations for Stationary Human Targets in the Initial Phase

(In meters)

Target position	Target-silhouette comparison	Pattern a		Pattern b		Pattern c	
		Mean	Standard deviation	Mean	Standard deviation	Mean	Standard deviation
240	Matched	264	15	281	10	304	15
325	Interpolated	348	14	348	11	350	24
358	Matched	403	32	403	19	389	21
460	Interpolated	485	11	497	14	513	14
492	Matched	486	15	509	28	514	46
561	Interpolated	554	34	612	28	591	15

TABLE A4

Measures of Percentage Error of Range Estimations for Stationary Human Targets in the Initial Phase

Target position, m	Target-silhouette comparison	Pattern a		Pattern b		Pattern c	
		Mean	Standard deviation	Mean	Standard deviation	Mean	Standard deviation
		Error, %					
240	Matched	8.7	5.0	14.4	3.0	20.8	4.1
325	Interpolated	6.4	3.7	6.6	2.9	7.0	3.6
358	Matched	10.6	7.4	11.0	4.2	7.6	5.0
460	Interpolated	5.1	2.5	7.4	2.4	9.8	2.1
492	Matched	-1.4	3.2	3.2	5.3	3.7	8.3
561	Interpolated	-1.5	6.1	8.1	4.3	5.1	2.5
Overall	Matched	6.0	7.4	9.5	6.3	10.7	9.4
Overall	Interpolated	3.1	5.4	7.4	3.1	7.3	3.3
Overall matched and interpolated		4.6	6.6	8.5	5.0	9.0	7.2

TABLE A5

Measures of Range Estimations for Fleeting Human Targets in the Initial Phase

(In meters)

Target position	Target-silhouette comparison	Pattern a		Pattern b	
		Mean	Standard deviation	Mean	Standard deviation
428	Matched	527	49	—	—
460	Interpolated	—	—	461	27
492	Matched	493	7	453	19

TABLE A6

Measures of Percentage Error of Range Estimations for Fleeting Human Targets in the Initial Phase

Target position, m	Target-silhouette comparison	Pattern a		Pattern b	
		Mean	Standard deviation	Mean	Standard deviation
		Error, %			
428	Matched	18.2	7.8	—	—
460	Interpolated	—	—	0.1	5.7
492	Matched	0.2	1.4	−8.7	4.6
Overall matched and interpolated		10.0	10.8	−4.3	6.6

TABLE A7

Range-Estimation Data for Vertically Arrayed Silhouettes in the Second Phase

(In meters)

Vertical array	Target position	Observer 1	2	3	4	5
Stationary Human Target						
In line	272	293	300	286	300	300
	300	300	332	348	348	319
	400	418	418	364	474	436
	436	504	496	536	511	474
	511	511	536	492	600	544
	536	600	610	610	610	610
Offset	272	300	300	300	300	300
	300	286	316	316	332	332
	400	492	429	382	455	455
	436	474	455	536	474	451
	511	507	502	492	511	511
	536	511	507	455	586	591
Fleeting Human Target						
In line	272	300	345	286	289	286
	300	464	324	316	316	381
	400	462	436	364	474	400
	436	429	418	382	436	455
	511	511	523	300	586	511

TABLE A8

Percentage Error of Range Estimations for Vertically Arrayed Silhouettes in the Second Phase

Vertical array	Target-silhouette comparison	Target position, m	Observer 1	2	3	4	5
			Error, %				
			Stationary Human Target				
In line	Interpolated	272	7.2	9.3	4.9	9.3	9.3
	Matched	300	0.0	9.6	13.8	13.8	6.0
	Interpolated	400	4.3	4.3	−9.9	15.6	8.2
	Matched	436	13.5	12.1	18.6	14.7	8.0
	Matched	511	0.0	4.7	−3.9	14.8	6.1
	Interpolated	536	10.7	12.1	12.1	12.1	12.1
Offset	Interpolated	272	−9.3	−9.3	−9.3	−9.3	−9.3
	Matched	300	−4.9	5.1	5.1	9.6	9.6
	Interpolated	400	18.7	6.8	−4.7	12.1	12.1
	Matched	436	8.0	4.2	18.6	8.0	3.3
	Matched	511	−0.8	−1.8	3.9	0.0	0.0
	Interpolated	536	−4.9	−5.7	−17.8	8.5	9.3
			Fleeting Human Target				
In line	Interpolated	272	9.3	21.2	4.9	5.9	4.9
	Matched	300	35.3	7.4	5.1	5.1	21.2
	Interpolated	400	13.4	8.2	−9.9	15.6	0.0
	Matched	436	−1.6	−4.3	−14.1	0.0	4.2
	Matched	511	0.0	2.3	−70.3	12.8	0.0

TABLE A9

Measures of Range Estimations for Stationary Human Targets In the Second Phase

(In meters)

Target position	Target-silhouette comparison	In line Mean	In line Standard deviation	Offset Mean	Offset Standard deviation
272	Interpolated	296	6	300	0
300	Matched	329	20	316	23
400	Interpolated	422	40	443	38
436	Matched	504	80	478	34
511	Matched	537	38	505	8
536	Interpolated	608	4	530	58

TABLE A10

Measures of Percentage Error of Range Estimations for Stationary Human Targets in the Second Phase

Target position, m	Target-silhouette comparison	In line		Offset	
		Mean	Standard deviation	Mean	Standard deviation
		Error, %			
272	Interpolated	8.0	2.0	9.3	0.0
300	Matched	8.6	5.8	4.9	5.9
400	Interpolated	4.5	9.3	9.0	8.7
436	Matched	13.4	3.9	8.4	6.1
511	Matched	4.3	7.1	−0.3	2.2
536	Interpolated	11.8	0.6	−2.1	11.3
Overall	Matched	8.8	6.5	4.4	6.0
Overall	Interpolated	8.1	5.9	5.4	9.4
Overall matched and interpolated		8.4	6.1	4.9	7.8

TABLE A11

Measures of Range Estimations for Fleeting Human Targets in the Second Phase

(In meters)

Target position	Target-silhouette comparison	In line	
		Mean	Standard deviation
272	Interpolated	301	27
300	Matched	360	64
400	Interpolated	427	46
436	Matched	424	29
511	Matched	486	109

TABLE A12

Measures of Percentage Error of Range Estimations for Fleeting Human Targets in the Second Phase

Target position, m	Target-silhouette comparison	In line: Mean (Error, %)	In line: Standard deviation (Error, %)
272	Interpolated	9.2	6.9
300	Matched	14.8	13.3
400	Interpolated	9.4	6.0
436	Matched	−3.2	6.8
511	Matched	3.8[a]	6.1[a]
Overall	Matched	5.2[a]	11.8[a]
Overall	Interpolated	9.3	6.1
Overall matched and interpolated		6.9[a]	9.9[a]

[a]Excluding 70.3 percent underestimated range error.

TABLE A13

Range-Estimation Data Measured from Film for Vertically Arrayed Silhouettes in the Second Phase

(In meters)

Vertical array	Target position	Observer 1	2	3	4	5
		Stationary Human Target				
In line	272	a	a	a	a	a
	300	300	308	358	358	312
	400	432	436	467	486	436
	436	507	505	507	509	461
	511	511	510	510	609	535
	536	601	612	612	612	612
Offset	272	291	287	294	291	291
	300	282	300	300	328	324
	400	495	428	360	459	456
	436	476	442	512	482	463
	511	510	511	436	511	595
	536	513	513	438	583	591
		Fleeting Human Target				
In line	272	310	377	262	272	438
	300	329	320	262	297	380
	400	477	428	360	463	394
	436	428	400	360	418	37[illegible]
	511	511	519	310	570	565

[a]Film not readable.

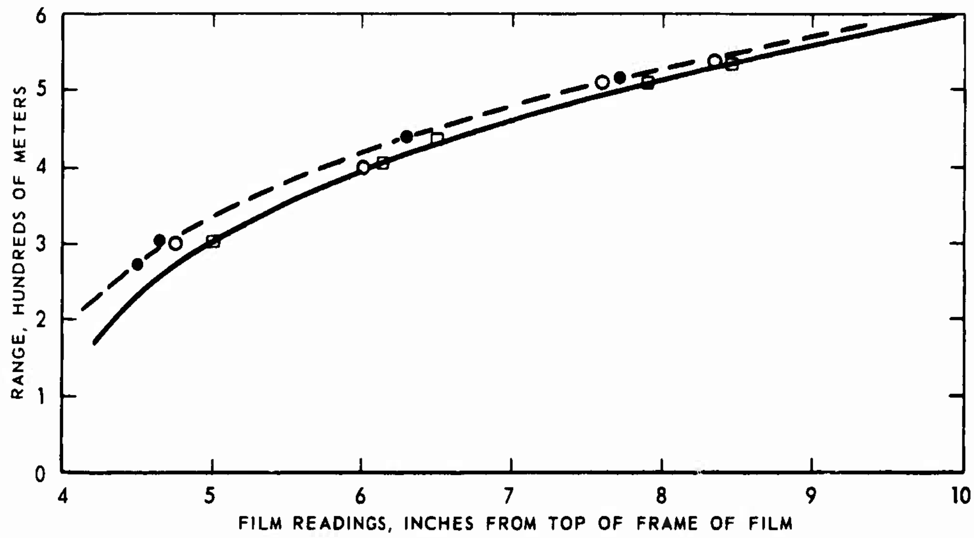


Fig. A1—Range Calibration of Vertical Reticle Silhouettes and Field Reference Markers by Film

□ Target stationary, in-line array

○ Target stationary, offset array

● Target fleeting, in-line array

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Appendix B

SIGNIFICANCE TESTS FOR HOMOGENEITY OF VARIANCE

Hartley's test for homogeneity of variance was employed for testing the equality of variances among the various groups of data for percentage error of range estimation. The tests consist of calculating the ratio of the maximum sample variance to the minimum sample variance in a set of k independent mean squares and comparison of the calculated value to corresponding tabular values of the approximate variance ratio distribution for statistical significance. Tables of percentage points of the ratio s^2_{max}/s^2_{min} are given in Pearson and Hartley's Biometrika Tables.[12]

If the calculated ratio of sample variances is numerically greater than the tabular value, a significant difference between or among variances is indicated. The following tests have been conducted using the 5 percent level of significance.

These tests indicate significant differences in the precision of the results of the various tests, arrays, patterns, target positions, and comparisons of match or interpolation.

INITIAL PHASE

Stationary human target

Pattern a

Among target positions $s^2_{max}/s^2_{min} = \frac{54.20}{6.01} = 9.02$

Tabular value (5 percent level) = 29.5

Between comparisons (match vs interpolate) $s^2_{max}/s^2_{min} = \frac{55.40}{29.67} = 1.87$

Tabular value (5 percent level) = 3.10

Significant difference indicated

Pattern b

Among target positions $s^2_{max}/s^2_{min} = \frac{28.16}{5.97} = 4.72$

Tabular value (5 percent level) = 29.5

Between comparisons (match vs interpolate) $s^2_{max}/s^2_{min} = \frac{39.07}{9.82} = 3.98$

Tabular value (5 percent level) = 3.10

Significant difference indicated

Pattern c

Among target positions $s^2_{max}/s^2_{min} = \frac{69.45}{4.27} = 16.26$

Tabular value (5 percent level) = 29.5

Between comparisons (match vs interpolate) $s^2_{max}/s^2_{min} = \frac{89.09}{10.76} = 8.28$

Tabular value (5 percent level) = 3.00
Significant difference indicated

Among patterns a vs b vs c $s^2_{max}/s^2_{min} = \frac{51.18}{25.25} = 2.03$

Tabular value (5 percent level) = 2.45

Fleeting human target

Pattern a

Between target positions $s^2_{max}/s^2_{min} = \frac{61.53}{2.09} = 29.44$

Tabular value (5 percent level) = 9.60
Significant difference indicated

Pattern b

Between target positions $s^2_{max}/s^2_{min} = \frac{32.96}{21.04} = 1.57$

Tabular value (5 percent level) = 9.60

Between patterns a vs b $s^2_{max}/s^2_{min} = \frac{117.67}{44.16} = 2.66$

Tabular value (5 percent level) = 4.03

SECOND PHASE

Stationary human target

Pattern d (in-line array)

Among target positions including all data $s^2_{max}/s^2_{min} = \frac{86.08}{0.39} = 220.72$

Tabular value (5 percent level) = 29.5
Significant difference indicated

Among target positions excluding farthest target-position (536 m) data points $s^2_{max}/s^2_{min} = \frac{86.08}{3.83} = 22.48$

Tabular value (5 percent level) = 25.2

Among match comparisons $s^2_{max}/s^2_{min} = \frac{49.84}{14.90} = 3.34$

Tabular value (5 percent level) = 15.5

Between interpolated comparisons (excluding 536 m positions) $s^2_{max}/s^2_{min} = \frac{86.08}{3.83} = 22.48$

Tabular value (5 percent level) = 9.60
Significant difference indicated

Between comparisons (match vs interpolate) excluding farthest target-position (536 m) data points $s^2_{max}/s^2_{min} = \frac{43.36}{42.80} = 1.01$

Tabular value (5 percent level) = 3.21

Pattern e (offset array)

Among target positions including all data $s^2_{max}/s^2_{min} = \frac{127.40}{0.0} = \infty$

<u>Significant difference indicated</u>

Among target positions excluding nearest target positions (272 m) with 0.0 variance $s^2_{max}/s^2_{min} = \frac{127.40}{0.57} = 223.51$

Tabular value (5 percent level) = 20.6

<u>Significant difference indicated</u>

Fleeting human target

Pattern d (in-line array)

Among target positions $s^2_{max}/s^2_{min} = \frac{1125.45}{0036.12} = 31.16$

Tabular value (5 percent level) = 25.2

<u>Significant difference indicated</u>

Among target positions excluding the underestimated range-error 70 percent data point $s^2_{max}/s^2_{min} = \frac{176.04}{36.12} = 4.87$

Tabular value (5 percent level) = 25.2

Among match comparisons $s^2_{max}/s^2_{min} = \frac{176.04}{37.38} = 4.71$

Tabular value (5 percent level) = 15.5

Between interpolated comparisons $s^2_{max}/s^2_{min} = \frac{47.97}{36.12} = 1.33$

Tabular value (5 percent level) = 9.6

Between comparisons (match vs interpolated) $s^2_{max}/s^2_{min} = \frac{140.32}{37.38} = 3.75$

Tabular value (5 percent level) = 3.83

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